

The direction of time in quantum field theory

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Abstract

The algebra of observables associated with a quantum field theory is invariant under the connected component of the Lorentz group and under parity reversal, but it is not invariant under time reversal. If we take general covariance seriously as a long-term goal, the algebra of observables should be time-reversal invariant, and any breaking of time-reversal symmetry will have to be described by the state over the algebra. In consequence, the modified algebra of observables is a presentation of a classical continuous random field.

First some mathematical preliminaries are necessary. Quantum field theory is presented at an elementary level in terms of an operator-valued distribution, $\hat{\phi}(x)$. That this is a distribution reflects the fact that $\hat{\phi}(x)$ is not itself an operator that we can associate with a measurement; for an operator, we have to *smooth* the quantum field by averaging, to obtain $\hat{\phi}_f = \int f(x)\hat{\phi}(x)d^4x$, where the *test function* $f(x)$ is generally taken to be a Schwartz space function, which is zero at infinity and smooth both in real space and, as $\tilde{f}(k)$, in Fourier space. There are notational, conceptual, and mathematical advantages to working with the *smeared* operators $\hat{\phi}_f$ instead of with the operator-valued distribution $\hat{\phi}(x)$, and we can always get back to operator-valued distributions, albeit improperly, by using Dirac delta functions. Routinely, $\hat{\phi}_f$ is expressed as the sum of non-observable creation and annihilation operators a_f^\dagger and a_f , $\hat{\phi}_f = a_f + a_{f^*}^\dagger$, where a_f and $a_{f^*}^\dagger$ are both complex linear in f to ensure that $\hat{\phi}_f$ is complex linear. The quantized Klein-Gordon field, for example (because it is the most elementary non-interacting quantum field), can be straightforwardly presented in terms of commutation relations between creation and annihilation operators[1],

$$[a_g, a_f^\dagger] = (f, g), \quad [a_f, a_g] = 0. \quad (1)$$

The Hermitian inner product (f, g) is manifestly Lorentz invariant, except for time reversal,

$$(f, g) = \hbar \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k^\mu k_\mu - m^2) \theta(k_0) \tilde{f}^*(k) \tilde{g}(k). \quad (2)$$

This fixes the algebraic structure of the quantized Klein-Gordon field operators. In particular, this construction ensures that the field $\hat{\phi}_f$ satisfies microcausality, so that $[\hat{\phi}_f, \hat{\phi}_g] = 0$ whenever the real-space supports of the functions f and g are space-like separated (in terms of operator-valued distributions, $[\hat{\phi}(x), \hat{\phi}(y)] = 0$ whenever x and y are space-like separated). **The $\theta(k_0)$ factor, however, which implements the requirement that the energy spectrum of the Hamiltonian operator must be positive, introduces an explicit direction of time into quantum field theory.** The Hamiltonian operator is the generator of time translations in quantum theory, and is required to be in the forward time-like direction, so this is not a surprise, but it deserves attention. Note that because of the connection with time translations and the positivity of the Hamiltonian, non-invariance under time-reversal transcends the straightforward free quantum field model that is described above.

The nature of this direction of time in quantum field theory has perhaps been less considered than it might have been because of the ways in which quantum field theory is usually presented. Especially curious is what happens if we change the inner product so that there is no explicit direction of time,

$$(f, g)_C = \frac{1}{2} \hbar \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k^\mu k_\mu - m^2) \tilde{f}^*(k) \tilde{g}(k) = \frac{1}{2} [(f, g) + (g^*, f^*)]. \quad (3)$$

In consequence of this choice, the quantum field $\hat{\phi}_f$ becomes classical — in fact a presentation of a continuous random field — in the sense that

$$[\hat{\phi}_f, \hat{\phi}_g] = [a_f, a_{g^*}^\dagger] + [a_{f^*}^\dagger, a_g] = \frac{1}{2} [(g^*, f) + (f^*, g) - (f^*, g) - (g^*, f)] = 0 \quad (4)$$

whatever functions we use for f and g . In this Hilbert space formalism, in other words, the choice of a direction of time is *the* difference between classical and quantum fields. If we take it that the algebra of observables *ought* to be invariant under the whole Lorentz group, including under the discrete parity and time-reversal symmetries — indeed, it ought to be diffeomorphism invariant, but that is for another day — then any lack of parity and time-reversal symmetry should be described by the state over the algebra of observables.

To consider the meaning of test functions and the projection to positive frequencies in the inner product, we will specialize to the vacuum sector, which is constructed by defining a vacuum state $|0\rangle$ as the zero eigenstate of all annihilation operators, $a_f|0\rangle = 0$ for all test functions. The vacuum sector, then, is the Fock space of states constructed by applying creation operators to the vacuum, $a_g^\dagger|0\rangle$, $a_{g_1}^\dagger a_{g_2}^\dagger|0\rangle$, ... (more abstractly, the Fock space can be constructed by using the GNS construction[2, Ch. 3]).

There are two ways in which this algebra of operators can be used in the vacuum sector. Most obviously, we could measure $\hat{\phi}_f$ in the vacuum state; for an ensemble of measurements in the vacuum state we would obtain a probability distribution with moments $\langle 0| \hat{\phi}_f^{2k} |0\rangle = \frac{(2k)!}{2^k k!} (f^*, f)^k$, $\langle 0| \hat{\phi}_f^{2k-1} |0\rangle = 0$, which correspond to a normal probability distribution with mean 0 and variance (f^*, f) ¹. This approach, however, is inappropriate for most real measurement apparatuses, which are tuned to give a zero response to the vacuum. A different approach, which is almost always used in some variant in quantum optics², is to use the projection operator

$$\hat{X}_f = \frac{a_f^\dagger |0\rangle \langle 0| a_f}{(f, f)}, \quad (5)$$

very often with an improper pure wave-number test function³. This kind of measurement asks whether a state *resonates* with the measurement apparatus; for example, in the vacuum state the moments of the probability distribution are all zero, signifying that we always observe 0; in the normalized state $|g\rangle = \frac{1}{\sqrt{(g, g)}} a_g^\dagger |0\rangle$ the moments of the probability distribution are all $p = \frac{1}{(g, g)(f, f)} |(f, g)|^2$, signifying that we observe 1 with probability p

¹ $\hat{\phi}_f$ is only an observable if $\hat{\phi}_f^\dagger = \hat{\phi}_f$, which requires that $f^* = f$ is real, so that $(f, f)_C = (f, f)$; for this observable vacuum classical and quantum probabilities coincide.

²The quantized electromagnetic field can be constructed exactly as above[3, 4], except that the inner product includes the components of bivector test functions $f_{\mu\nu}$ and $g_{\mu\nu}$,

$$(f, g)_{EM} = \hbar \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k_\alpha k^\alpha) \theta(k_0) \tilde{f}_{\mu\beta}^*(k) k^\mu k^\nu \tilde{g}_\nu^\beta(k).$$

The algebraic structure is thus identical above the level of the inner product, but the geometrical structure in space-time that is expressed by the inner product is different.

³We cannot use the (f, f) normalization constant to construct a true projection operator for a pure wave-number test function. (f, f) is not defined for a delta function in Fourier space, a pure frequency in a single direction that is evenly distributed over all of space-time. Note that the commutator $[\hat{X}_f, \hat{X}_g]$ is generally non-zero when the supports of f and g are space-like separated, so quantum optics formalisms which use this or similar operators are not causally local in this sense. Nor is \hat{X}_f linear in f .

and 0 with probability $1 - p$. Sometimes the measurement apparatus will resonate, sometimes it won't, depending on how closely parallel the test functions g and f are in terms of the inner product that defines the algebraic structure. Quantum optics has constructed many useful states and measurement operators that are used to model experiments, which will not be further rehearsed here.

Every construction of an observable that is possible in quantum field theory is also possible for a classical continuous random field, using the classical inner product $(f, g)_C = \frac{1}{2} [(f, g) + (g^*, f^*)]$ instead of using the quantum inner product (f, g) ; superpositions and interference are just as possible for continuous random fields as for quantum fields. What, then, is the difference between the classical and the quantum inner products? Firstly, the difference between the quantum and classical inner products, $(f, g) - (f, g)_C = \frac{1}{2} [(f, g) - (g^*, f^*)]$, is zero when the supports of f and g are space-like separated. Additionally, there is precisely a factor of two between the quantum and classical inner products if classical modeling uses only test functions that are restricted to positive frequencies (a choice that results in the *analytic signal* in classical signal analysis, so we may perhaps use the name *analytic test function*). With test functions used in classical models restricted to positive frequency, quantum optics and a classical continuous random field version of quantum optics are operationally identical, albeit with an inessential factor of 2^4 . In effect, the continuous random field exploits more degrees of freedom than the corresponding quantum field theory, and has the same functional dependence on the common degrees of freedom, so it can accommodate empirical data at least as well. Note that it is a commonplace in classical signal analysis that the perfect measurement of signal frequency is incompatible with the measurement of the signal for only a finite time, so that — for example, because signal analysis is a large subject — the Wigner function is a common tool in classical signal analysis[5]. I have discussed the differences, similarities, and relationships between the classical and quantum theories of measurement and their algebras of observables, from a field theory point of view, elsewhere[1, 6, 3, 7]⁵.

⁴Measurements constitute a set of constraints on the ansätze that are chosen as models for a given set of experiments. If the constraints are satisfiable by f , they are also satisfiable by a constant multiple of f .

⁵Of related interest, Hobson[8, 9] has recommended using ideas from quantum field theory when motivating quantum mechanics at the undergraduate level. In Hobson's approach, however, fields have a particle aspect that causes discrete events, whereas I prefer to understand events as the result of resonances of the field with the carefully tuned thermodynamic properties of experimental apparatus that are not point-like.

There is a significant sense in which quantum field theory is overconstrained by the restriction to positive frequency: there are no known interacting quantum field theories on Minkowski space. In contrast, with the introduction of negative frequencies a large class of interacting continuous random field theories can be constructed[3], following an approach that was tried but abandoned for quantum fields in the 1960s. Hence, there is both a significant mathematical advantage and a significant conceptual advantage to using classical continuous random field models consistently in Physics. Of course there may be other constraints that have not yet been considered that will make continuous random fields either impossible or impractical, or simply on balance not attractive to Physicists, but note that Bell inequalities are not more problematic than they are for quantum fields[10, 7].

If we eliminate the direction of time from the algebra of observables, there will presumably be a significant breaking of time invariance in the states we construct, for we know that it is most often possible to model experiments in Physics using only fields, without having to resort to their time-reversed anti-fields⁶. A continuous random field formalism effectively has no anti-fields because the algebra of observables is already time-reversal invariant.

The immediate consequences for quantum field theory of enforcing time-reversal invariance of the algebra of observables are extreme: instead of using quantum field models, we use continuous random field models, and we can use Lie fields to express non-Gaussian vacuum correlations[3], instead of having to resort to renormalization. The Lie field approach that is made available when we require time-reversal invariance of the algebra of observables results in a reconceptualization of Physics that goes far beyond the Nature of Time. The Lie field approach, however, is essentially an empiricist intermediary for future theories, because only correlations are explicitly modeled; causality, which is an essential part of the explanatory and predictive power of a theory but cannot be directly measured in the quantum mechanical world of discrete measurement events, is only emergently part of a continuous random field model.

References

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⁶Since “anti-field” is not to my knowledge an existing terminology, please substitute “anti-particle” if you cannot yet give up particle language.

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